

24018

B.Tech. 2nd Semester F. Scheme Examination,

May-2012

MATHEMATICS - II

Paper - Math-102-F

Time allowed : 3 hours]

[Maximum marks : 100

Note : Question No. 1 is compulsory. Attempt total five questions with selecting one question from each unit. All questions carry equal marks.

1. (a) Find the value of constant 'a' such that

$\vec{A} = (ax + 4y^2z) \mathbf{i} + (x^3 \sin z - 3y) \mathbf{j} - (e^x + 4 \cos x^2y) \mathbf{k}$ is solenoidal.

- (b) Show that, if $R = A \sin \omega t + B \cos \omega t$, where A, B, ω are constants, then :

$$\frac{d^2R}{dt^2} = -\omega^2 R \text{ and } R \times \frac{dR}{dt} = -\omega \times R$$

- (c) Solve $p(1+q) = qz$

- (d) Find Laplace transformation of $e^{-4t} t \cos 2t$.

- (e) Solve $\frac{\partial^2 z}{\partial y \partial x} = xy$

- (f) Solve $\frac{d^2y}{dx^2} + \frac{dy}{dx} = \cos 2t$

- (g) Find the inverse Laplace transformation of :

$$\frac{s+3}{s^2+6s+13}$$

- (h) Solve $(2x^3 - xy^2 - 2y + 3) dx - (x^2y + 2x) dy = 0$

Unit - I

2. (a) Find the directional derivative of the function $f(x, y, z) = x^2yz + 4xz^2$ at the point $(1, -2, -1)$ in the direction of the vector $2\hat{i} - \hat{j} - 2\hat{k}$.

- (b) Prove that :

$$(i) \quad \nabla \times (F \times G) = (\nabla \cdot G) F - (\nabla \cdot F) G + (G \cdot \nabla) F - (F \cdot \nabla) G$$

$$(ii) \quad \nabla^2 f(r) = f''(r) + \frac{2}{r} f'(r)$$

3. (a) Verify Stoke's theorem for the vector field $\vec{F} = (2x - y)\hat{i} - yz^2\hat{j} - y^2z\hat{k}$ over the upper half surface of $x^2 + y^2 + z^2 = 1$ bounded by its projection on the xy -plane.

- (b) Apply Green's theorem to evaluate :

$\oint [(y - \sin x) dx + \cos x dy]$ where C is the plane triangle enclosed by the lines

$$y = 0, x = \frac{\pi}{2} \text{ and } y = \frac{2}{\pi} x$$

Unit - II

4. (a) Solve the following differential equation :
- $$(xy^2 + 2x^2y^3) dx + (x^2y - x^3y^2) dy = 0$$
- (b) A constant e.m.f. E volts is applied to a circuit containing a constant resistance R ohms in series and a constant inductance L henries. If the initial current is zero, show that the current builds up to half its theoretical maximum is $(L \log 2) / R$ seconds.
5. (a) By the method of variation of parameters, solve

$$\frac{d^2y}{dx^2} + 4y = \tan 2x$$

- (b) Solve the equation :

$$(1+x)^2 \frac{d^2y}{dx^2} + (1+x) \frac{dy}{dx} + y = 4 \cos \log (1+x)$$

Unit - III

6. (a) Using Convolution theorem, find inverse Laplace transformation of :

$$\frac{s}{(s^2 + a^2)^2}$$

- (b) Solve

$$\frac{d^2x}{dt^2} + 9x = \cos 2t, x(0) = -1; x'(0) = 2$$

using Laplace transform

7. (a) Find the Laplace transform of :

(i) $te^{-4t}\sin 3t.$

(ii) Evaluate $\int_0^{\infty} e^{-3t} \sin t dt$

(b) Find inverse Laplace transformation of :

$$(se^{-s/2} + \pi e^{-s}) / (s^2 + \pi^2).$$

Unit-IV

8. (a) Solve the following differential equation

$$(y + z) p - (z + x) q = (x - y)$$

(b) Solve the equation by Charpit's method :

$$(p^2 + q^2) x = pz$$

9. (a) Form the partial differential equation by eliminating the function :

$$f(x + y + z, x^2 + y^2 + z^2) = 0$$

(b) A string is stretched and fastened to two points $x = 0$ and $x = l$. Motion is started by displacing the string in the form $y = a \sin \left[\frac{\pi x}{l} \right]$ from which it is released at time $t = 0$. Show that the displacement is given by :

$$y(x, t) = a \sin \left(\frac{\pi x}{l} \right) \cos \left(\frac{\pi ct}{l} \right)$$